Quantum Chaos in Cooperative Atoms and Resonant Radiation Systems

レーザー発振における量子カオス

Toshihiko OHNO
Kurume Institute of Technology

1. Introduction

Quantum theoretical treatment of chaotic oscillation of laser is considered by using the model of the systems of kicked spins which are interacting with the single mode quantized radiation field represented by harmonic oscillater systems. Classical treatment of radiation field is also considered , beforehand, in order to consider the correspondings between quantized field and classical ones.

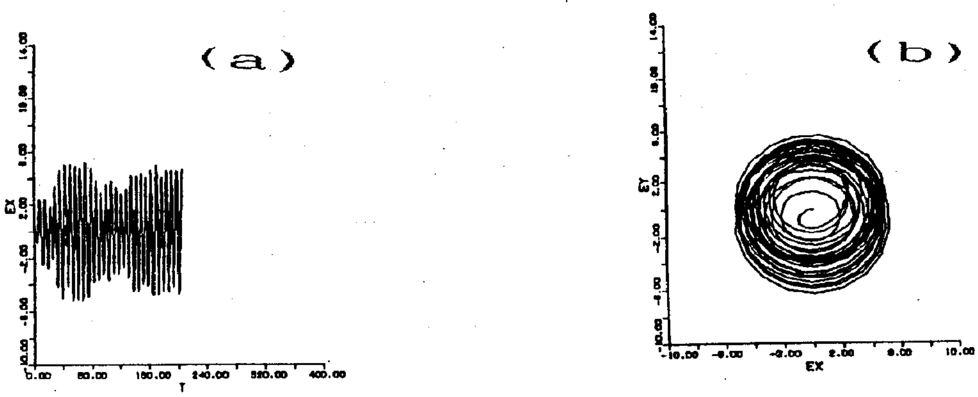
2.Semiclassical systems

Semiclassical equations of spin systems are represented by coupled bloch equations, where spins are kicked periodically and coupled by the classical electric field. These are as folows;

$$\frac{d}{dt}\langle S^+\rangle = i\,\omega_a\langle S^+\rangle(\cos^2\Omega_T n-\sin^2\Omega_T n)-2\langle S_z\rangle\sin\Omega_T n\cos\Omega_T n-2g(a-a^+)\langle S^z\rangle$$

$$\frac{d}{dt}\langle S^z\rangle=\omega_a(\langle S^+\rangle+\langle S^-\rangle)\sin\Omega_T n\cos\Omega_T n-g(a-a^+)(\langle S^+\rangle-\langle S^-\rangle)$$

Fig.1 shows the trajectory of electric field, which behaves like the spin systems because field and spins are coupled directly. Magnitude of coupling and detuning of resonant frequencys of spin and field are the parameters which determine the chaotic osilla- tion of systems.



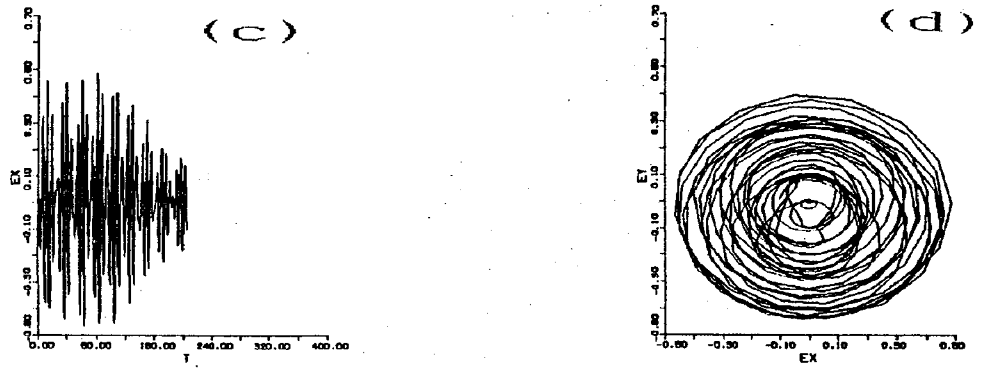


Fig.1 Semiclassical field for the case of (a),(b)g=0.5 (strong coupling),(c),(d)g=0.1.

3.Quantum theory

Quantum systems corresponding to the semiclassical ones, is described by the density oprator which is also described by the distribution function for atomic or field coherent states. Those formulation are completed according to master equations of spins and field systems. We obtains the equations of distribution of atomic(spin) or field expectation variables as follows;

$$i\hbar\frac{\partial}{\partial t}\rho=[\exp(-\int^{t}_{0}H_K/i\hbar dt)H\exp(\int^{t}_{0}H_K/i\hbar dt),\ \rho]$$

$$H_K=\Sigma\,\Omega_T\,\delta(t/T-n)(S^{+}+S^{-})$$

4.Discussion

Fig.2 shows the trajectory of expectation values of field variables, which were obtained by using the expansion coefficient of distribution function. This shows that chaotic behavior of field observed in semiclassical systems, disappears for the case of corresponding quantum systems. On the other hand, photon number variables have chaotic motion, which described by second order coefficients of expansion. The reason of the difference between semiclassical and quantum ones is clear when the form of each equations are compared. In the semiclassical case the field variables are coupled to the spin variables in phase, not in the case of quantum theory, where photons are emitted in the form of descrete energy.

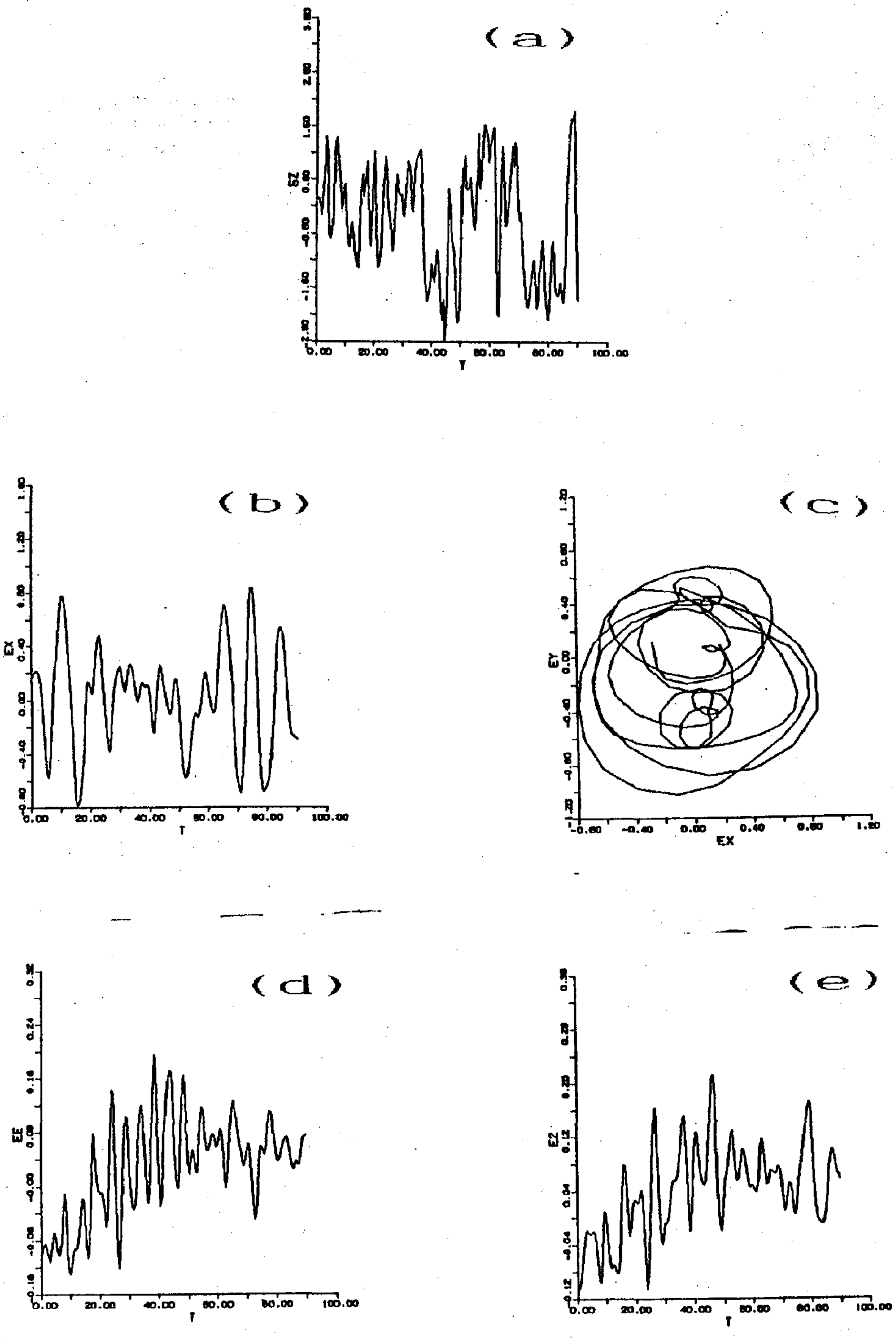


Fig.2 Spin and Field by quantum theory. (a)spin variable.(b).(c) electric field, and (d).(e) its second order correlations.